

Willard Evan Bleick

MINIMAL-TIME SHIP ROUTING.

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W. E. Bleick and F. D. Faulkner

Professors of Mathematics and Mechanics

RESEARCH PAPER NO. 46

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Minimal-Time Ship Routing

W. E. BLEICK and F. D. FAULKNER

U. S. Naval Postgraduate School, Monterey, Calif.

ABSTRACT

A recent theory of minimal-time ship routing through time-dependent ocean wave height and direction fields is put to a numerical test by using a series of semidaily analyses furnished by the U. S. Navy Fleet Numerical Weather Facility. The interpolations and integrations required are found to be feasible. A resume of the theory is given.

1. Introduction

Haltiner, Hamilton and Arnason (1962) gave a relaxation method solution to the problem of minimal-time routing of ships through ocean wave height and direction fields dependent on the ship location coordinates only. The theory has been extended by Faulkner (1963) to the case where the wave height and direction depend on time also. The present paper confronts this time-dependent theory with actual wave height and direction analyses from the files of the U. S. Navy Fleet Numerical Weather Facility, and reports on practical problems which had to be solved in a test of the theory.

2. Polar velocity diagram

A basic ingredient of the theory is the polar diagram of Fig. 1, giving the ship velocity v as a function of the angle θ between the ship's heading and the wave direction. A diagram of this kind must be specified for each wave height H . The points L, M and N on the diagram correspond to the ship speed v_h in head waves, v_b in beam waves, and v_f in following waves. Empirical curves for these three speeds as functions of wave height H are available in the pioneer

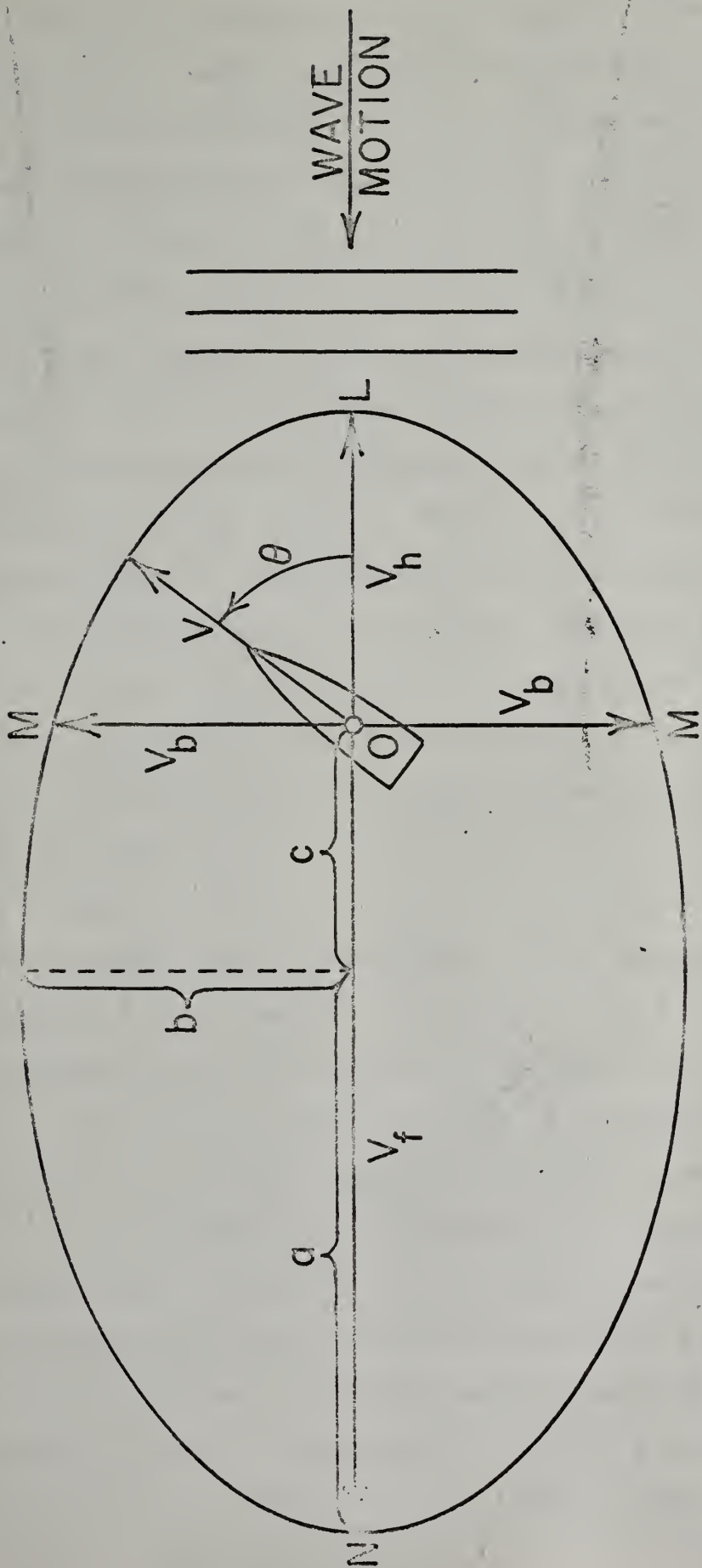


Fig. 1. Polar velocity diagram.

work of James (1957). His P2-S2-R2 ship type curves, shown in Fig. 2, have been chosen for use here. They have the appearance of being arcs of hyperbolas, at least approximately. This was confirmed when a least squares analysis showed that all three of v_h , v_b and v_f can be represented closely as functions of H by the hyperbolic arc

$$c_1 + c_2 H - [(c_3 + c_4 H)^2 + (c_1 - c_0)^2 - c_3^2]^{\frac{1}{2}} \quad (1)$$

where c_0 is the point common to all three speeds when $H=0$. The other four constants are related to the asymptotes $(c_2 + c_4)H + c_1 + c_3$ and $(c_2 - c_4)H + c_1 - c_3$. It was decided to construct the polar velocity diagram by fitting an ellipse to the points L, M and N. This resulted in semi-principal axes $a = (v_h + v_f)/2$, $b = av_b / (v_h v_f)^{\frac{1}{2}}$, and a distance to the eccentric pole 0 given by $c = (v_f - v_h)/2$. Note that the pole 0 is not to be construed as a focus of the ellipse. A further least squares analysis showed that the semiaxes a and b are closely representable also by hyperbolic functions of the form of (1), but not c which must be calculated as $c = (v_f - v_h)/2$.

3. Coordinate system

The ocean wave height and direction data of the semidaily Fleet Numerical Weather Facility analyses are presented in a south-polar stereographic projection of the northern hemisphere upon a plane passing through the circle of 60° North latitude. A rectangular coordinate system is set up in this projecting plane with the Ox and Oy axes parallel to the projections of the meridians of 10° and 100° East longitude respectively. A 62 by 62 grid is constructed using these axes with $x=y=31$ defining the projection of the North pole. The mesh distance between grid lines corresponds to a distance of 381 km at 60° North latitude where the projection is true. The radius of the equator's projection is 31.205 mesh units. The map scale factor m ,

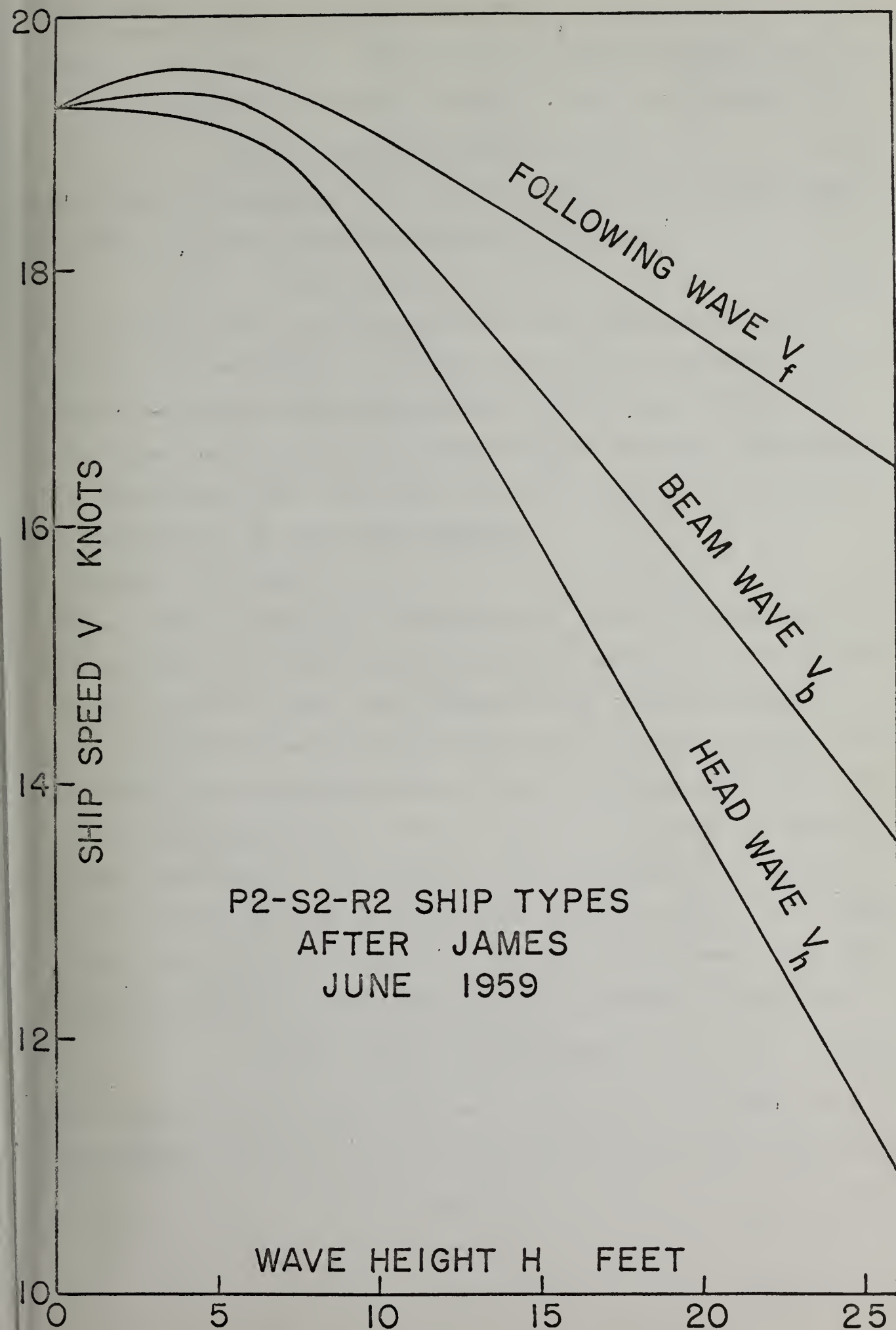


Fig. 2. Ship speed in following, beam and head waves.

defined as the ratio of a differential distance in the Oxy plane to the corresponding differential distance on the earth's surface, is

$$m=[973.75+(x-31)^2+(y-31)^2]/1043.6 \quad (2)$$

Let x, y be the coordinates of a ship's projection in the Oxy plane at time t . Then the projected speed of the ship is

$$V(x, y, t, \theta) = v(H, \theta)m \quad (3)$$

where $v(H, \theta)$ is the actual geographical ship speed of the polar velocity diagram of Fig. 1, and where $H(x, y, t)$ is obtained by interpolation in the Fleet Numerical Weather Facility grid wave height data. Since the stereographic projection is a conformal transformation preserving angles and their senses, the angle θ is the same in the Oxy plane as on the earth's surface.

4. Resume of the theory

Fig. 3 shows a ship at the point (x, y, t) in the stereographic projection plane on a route from fixed initial point A at $t=0$ to fixed terminal point B at $t=T$. The elliptical polar velocity diagram for $V=mv$ is plotted at this point by interpolation in the semidaily wave height $H(x, y, t)$ and wave direction $K(x, y, t)$ grid values of the Fleet Numerical Weather Facility. The special type of interpolation required is discussed in the Appendix. The direction of the ship's velocity vector $\underline{V} = i\dot{x} + j\dot{y}$ is the control angle p , which is to be chosen at each point so as to minimize the transit time T from A to B. The equations of motion of the ship's projection in the Oxy plane are

$$\varphi_1 = \dot{x} - V \cos p = 0, \quad \varphi_2 = \dot{y} - V \sin p = 0 \quad (4)$$

where $V = |\underline{V}| = V(x, y, t, p)$. The problem of minimizing T is equivalent to the Lagrange calculus of variations problem of requiring the integral

$$I = \int_0^T (1 + \lambda \varphi_1 + \mu \varphi_2) dt \quad (5)$$

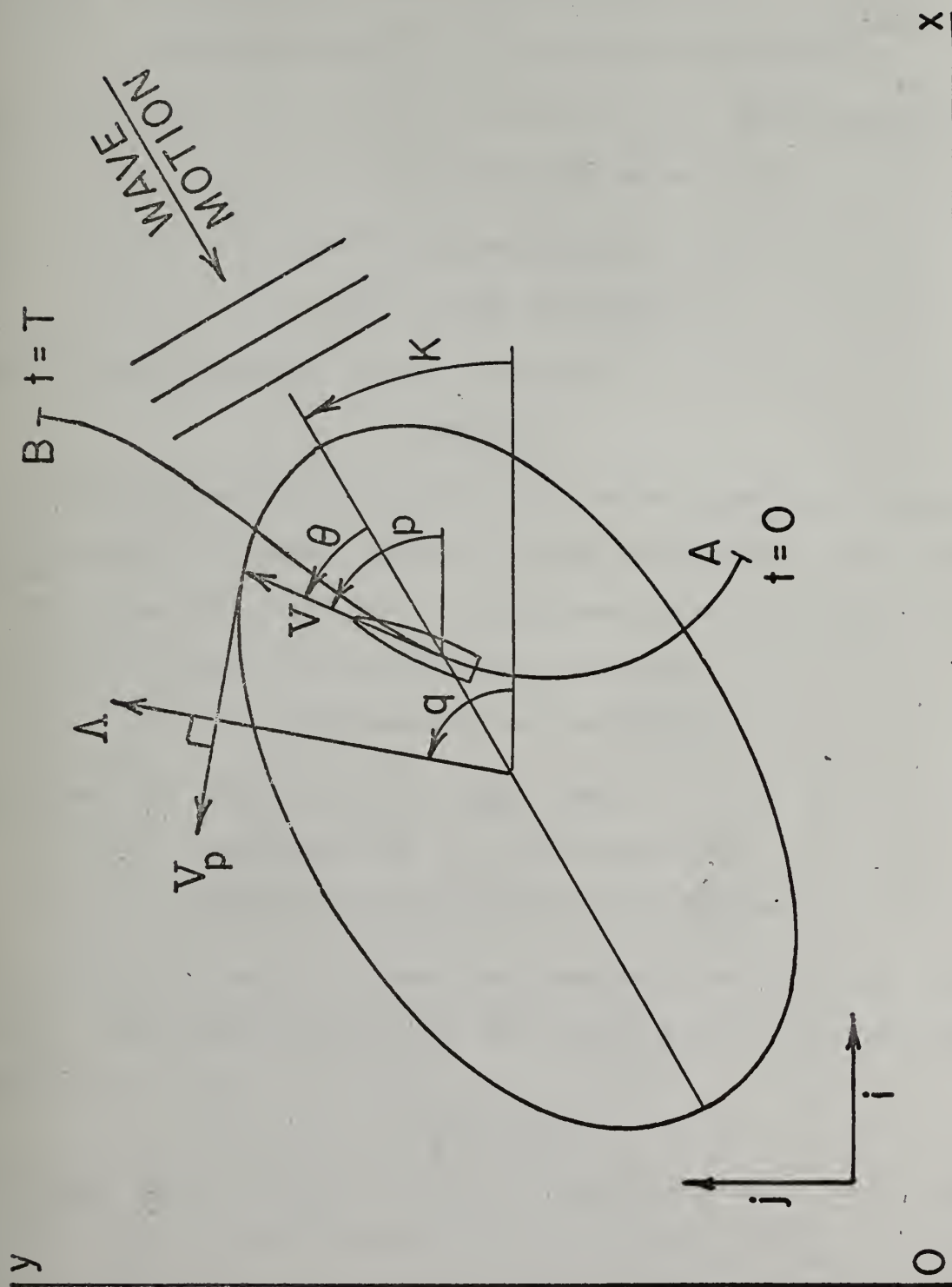


Fig. 3. Ship motion in stereographic Oxy plane.

to be stationary, where $\lambda(t)$ and $\mu(t)$ are continua of Lagrangian multipliers. Let the time at the fixed terminal point B be varied to $T+\Delta T$. The vanishing first variation of I is

$$\delta I = \Delta T + [\lambda \delta x + \mu \delta y]_0^T - \int_0^T (\varphi_3 \delta x + \varphi_4 \delta y + \varphi_5 \delta p) dt = 0. \quad (6)$$

The coefficients of $\delta x, \delta y, \delta p$ in $\delta I = 0$ give the Euler equations (7), (8) and (9), consisting of the adjoint equations

$$\varphi_3 = \dot{\lambda} + (\lambda \cos p + \mu \sin p) V_x = 0, \quad (7)$$

$$\varphi_4 = \dot{\mu} + (\lambda \cos p + \mu \sin p) V_y = 0, \quad (8)$$

and the scalar product control equation

$$\varphi_5 = \underline{\wedge} \cdot \underline{V}_p = 0, \quad (9)$$

where the adjoint vector $\underline{\wedge} = i\lambda + j\mu$, and where $\underline{V}_p = \partial V / \partial p = i(V_p \cos p - V \sin p) + j(V_p \sin p + V \cos p)$ is the tangent vector to the polar velocity diagram of Fig.3. Eq. (9) implies the orthogonality of $\underline{\wedge}$ and \underline{V}_p as shown in Fig.3. Eq.(9) may be written also in the form

$$p = \arctan(\mu/\lambda) + \arctan(V_p/V). \quad (10)$$

The fixed end points A and B imply that

$$dx(0) = \delta x(0) = 0, \quad dy(0) = \delta y(0) = 0, \quad (11)$$

$$dx(T) = (\dot{x}\Delta t + \delta x)_T = 0, \quad dy(T) = (\dot{y}\Delta t + \delta y)_T = 0. \quad (12)$$

Use of Eqs.(11) and (12) makes the remaining terms of (6) proportional to ΔT , whose coefficient gives the scalar product transversality condition

$$(\underline{\wedge} \cdot \underline{V})_T = 1 > 0 \quad (13)$$

meaningful for sign only because of the homogeneity of (7) and (8).

Eq.(13) implies that the angle $(q-p)$ between \underline{V} and $\underline{\wedge}$ is acute, as shown in Fig.3. A further implication of (10) and (13) is that the quadrant of p is such that

$$\cos p = (\lambda V - \mu V_p) / \wedge R, \quad \sin p = (\lambda V_p + \mu V) / \wedge R, \quad (14)$$

where $\Lambda = |\Lambda| = (\lambda^2 + \mu^2)^{\frac{1}{2}}$ and $R = |V_p| = (V^2 + V_p^2)^{\frac{1}{2}}$.

The simultaneous numerical integration of (4), (7), (8) and (14) is carried out together with a Newton-Raphson iteration as follows: Let λ_1, μ_1 and λ_2, μ_2 be two linearly independent solutions of the adjoint Eqs. (7) and (8) corresponding to the columns of the matrix

$$E(t) = \begin{bmatrix} \lambda_1 & \lambda_2 \\ \mu_1 & \mu_2 \end{bmatrix} \quad (15)$$

where $E(0) = I$ is the identity matrix. The λ, μ of (14) are taken as the linear combinations $\lambda = \lambda_1 \cos \alpha + \lambda_2 \sin \alpha$ and $\mu = \mu_1 \cos \alpha + \mu_2 \sin \alpha$. The variation δp is found by total differentiation of (10) to be

$$\delta p = R |E| \delta \alpha / \Lambda^2 (V^2 + 2V_p^2 - VV_{pp}) \quad (16)$$

where $|E|$ is the determinant of E . Assume that a solution of the ship motion Eqs. (4) has been found, corresponding to (7), (8) and (14) for some value of α , which falls short of the fixed end point B at $t=T$ by the coordinate differences $\Delta x(T)$ and $\Delta y(T)$. Using this solution and holding T fixed, find the variation of the vanishing matrix integral

$$\int_0^T [\varphi_1, \varphi_2] E(t) dt = 0. \quad (17)$$

Since the columns of $E(t)$ satisfy the adjoint Eqs. (7) and (8), one obtains the 1×2 matrix equation

$$[\delta x, \delta y]_T E(T) = \int_0^T [(V_p \cos p - V \sin p), (V_p \sin p + V \cos p)] E \delta p dt. \quad (18)$$

Substitution from (14) and (16) into (18) gives

$$[\delta x, \delta y]_T = [-\mu, \lambda]_T J \delta \alpha \quad (19)$$

where

$$J = -\frac{1}{|E(T)|} \int_0^T R^3 |E|^2 dt / \Lambda^3 (V^2 + 2V_p^2 - VV_{pp}). \quad (20)$$

Now vary the terminal time from T to $T + \Delta T$ and substitute

$$[\delta x, \delta y]_T = [\Delta x, \Delta y]_T - [\dot{x}, \dot{y}]_T \Delta T \quad (21)$$

into (19) to obtain the Newton-Raphson equations

$$\begin{aligned}\dot{x}(T)\Delta T - J\mu(T)\delta\alpha &= \Delta x(T) \\ \dot{y}(T)\Delta T + J\lambda(T)\delta\alpha &= \Delta y(T)\end{aligned}\tag{22}$$

for the determination of ΔT and $\delta\alpha$ on a varied trajectory which attempts to correct the errors $\Delta x(T)$ and $\Delta y(T)$. The iteration to successive varied trajectories is continued until the terminal errors are acceptable. A suitable initial guess for the angle α is the inclination angle of the straight line from A to B.

5. Numerical example

Ten successive semidaily analyses of wave height and direction, starting at 06Z on 4 May 1963, were furnished by the Fleet Numerical Weather Facility. The maximum speed of the chosen P2-S2-R2 ship type is 19.6 knots. This combination of data precluded a trip of great length. It was decided to select an area of continued extreme wave height for the example. Such an area was found centering at 30° North latitude and 162° East longitude. Figs. 4, 5 and 6 show two computed minimal-time ship tracks in the area, with contours of wave height in feet and wave direction arrows. The arc traversed by the ship during the 6 hours preceding and/or the 6 hours following the time of each Figure is shown as a dashed curve. The minimal-time track AB required 2.488 days with a 3.0% saving over the geodesic track. The minimal-time track CD required 2.656 days with a 1.3% saving over the geodesic track. The severity of the sea conditions in the area preclude any more spectacular saving. The highly non-analytic nature of the wave height in the area was found to affect the convergence of the Newton-Raphson iteration of (22). It was found necessary to halve the values of ΔT and $\delta\alpha$ in order to avoid a divergent oscillation. Resort to this stratagem was found to be unneces-

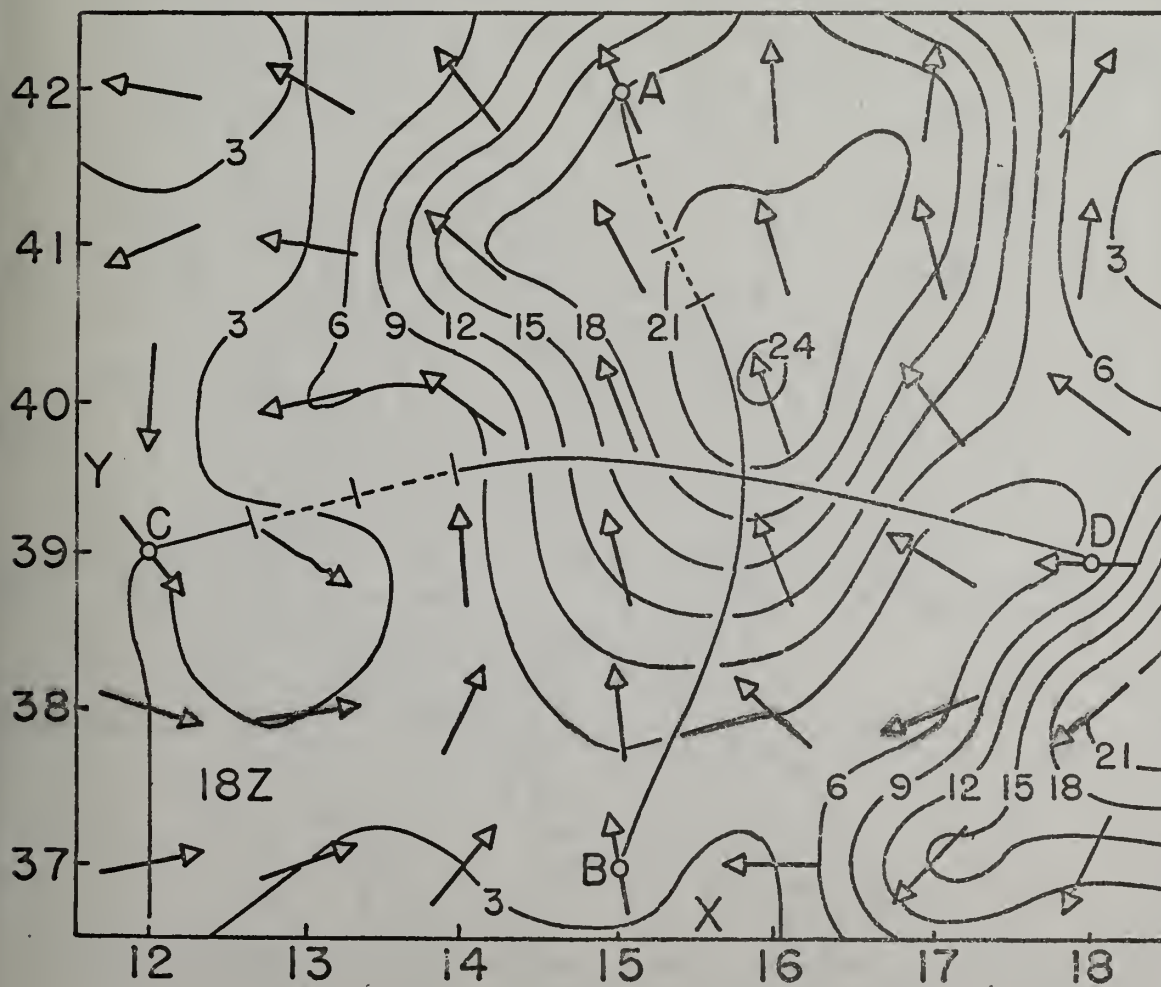
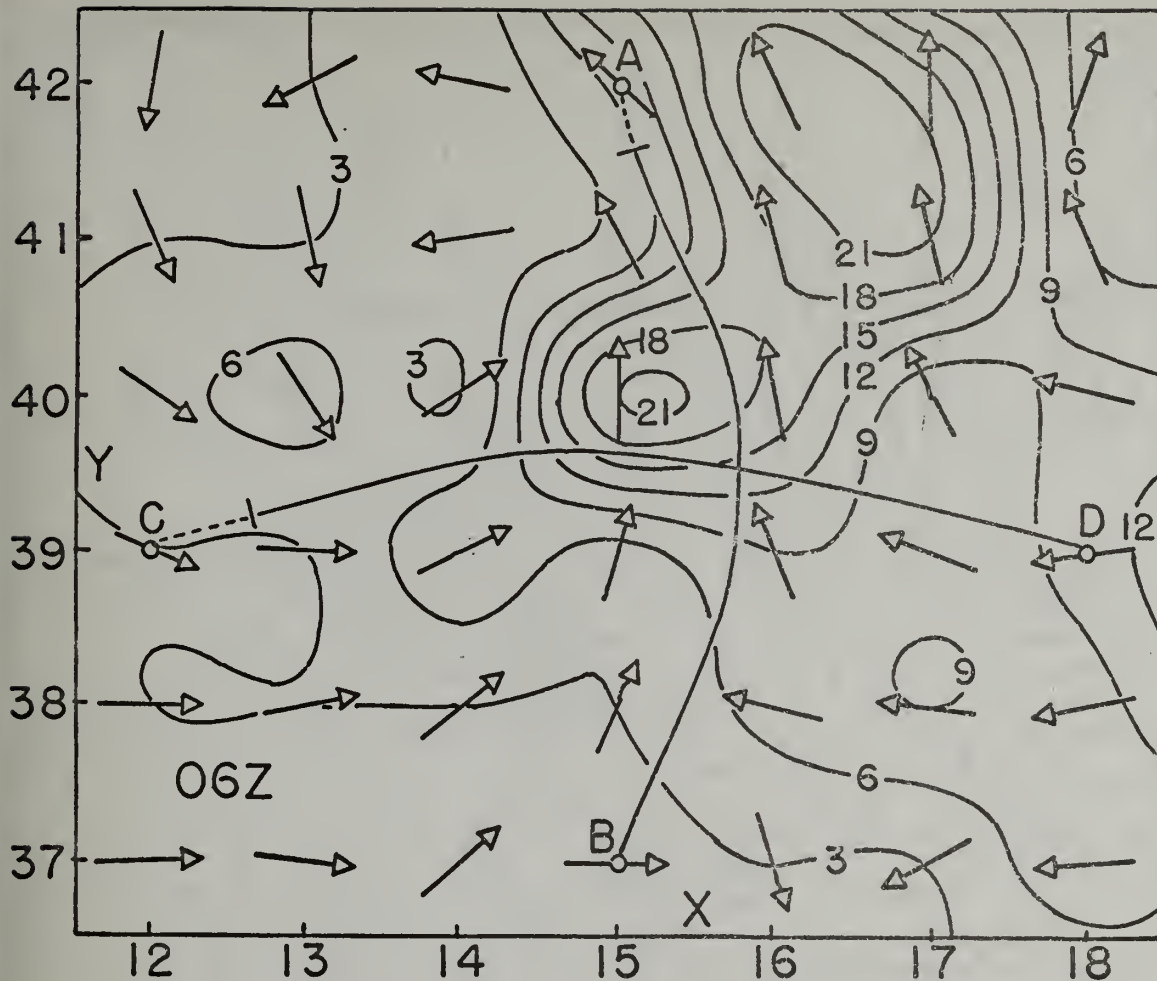


Fig. 4. Sea conditions at 06Z and 18Z on 4 May 1963.

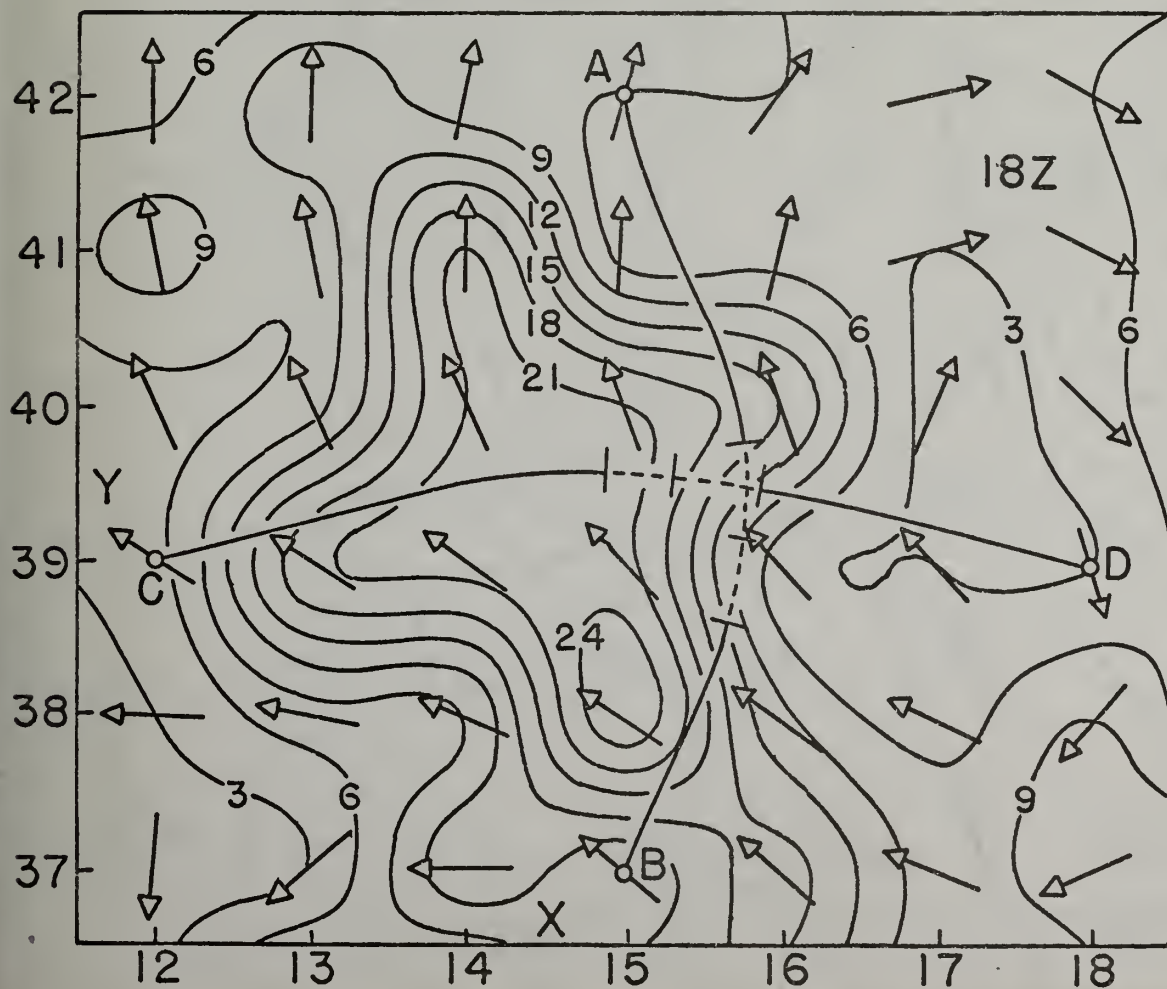
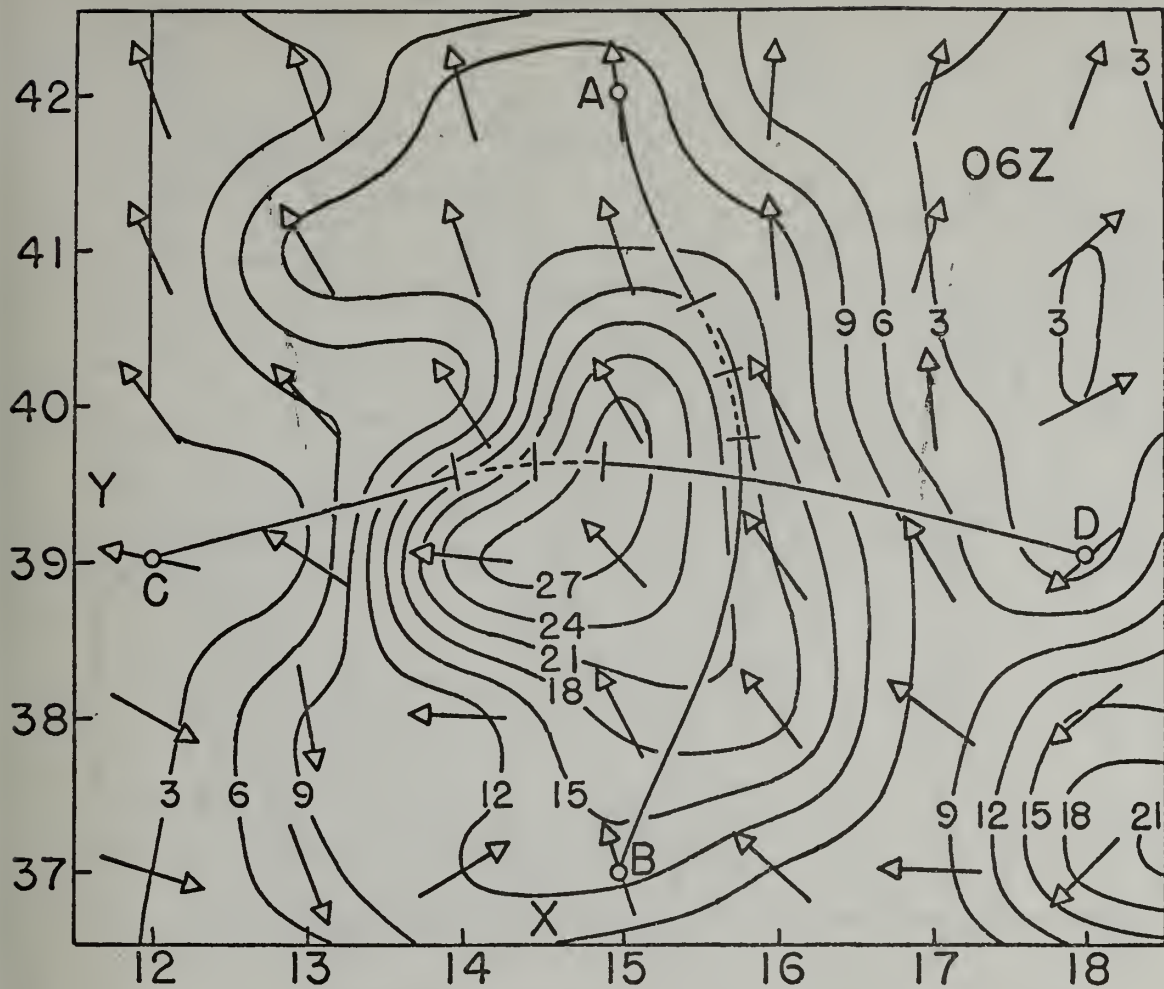


Fig. 5. Sea conditions at 06Z and 18Z on 5 May 1963.

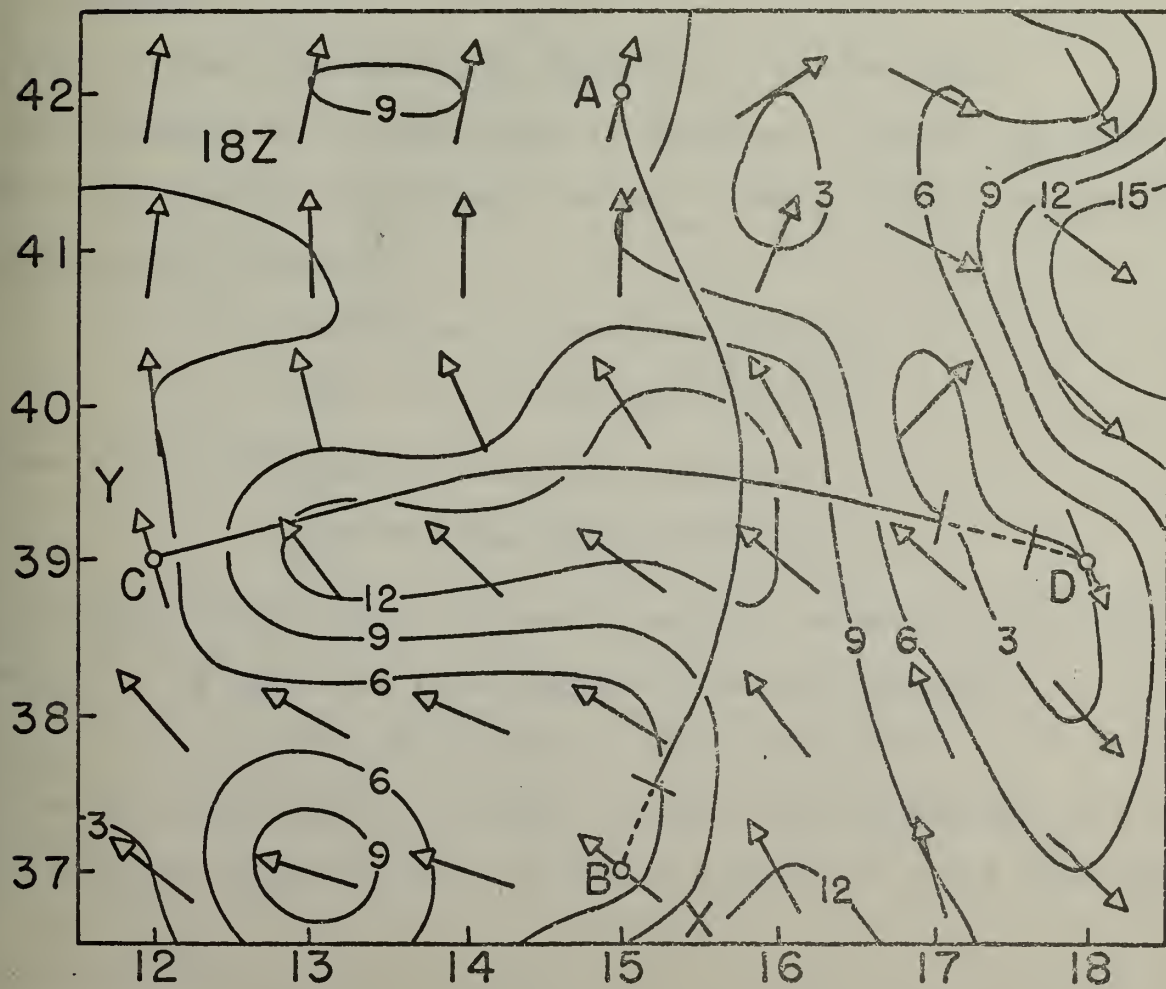
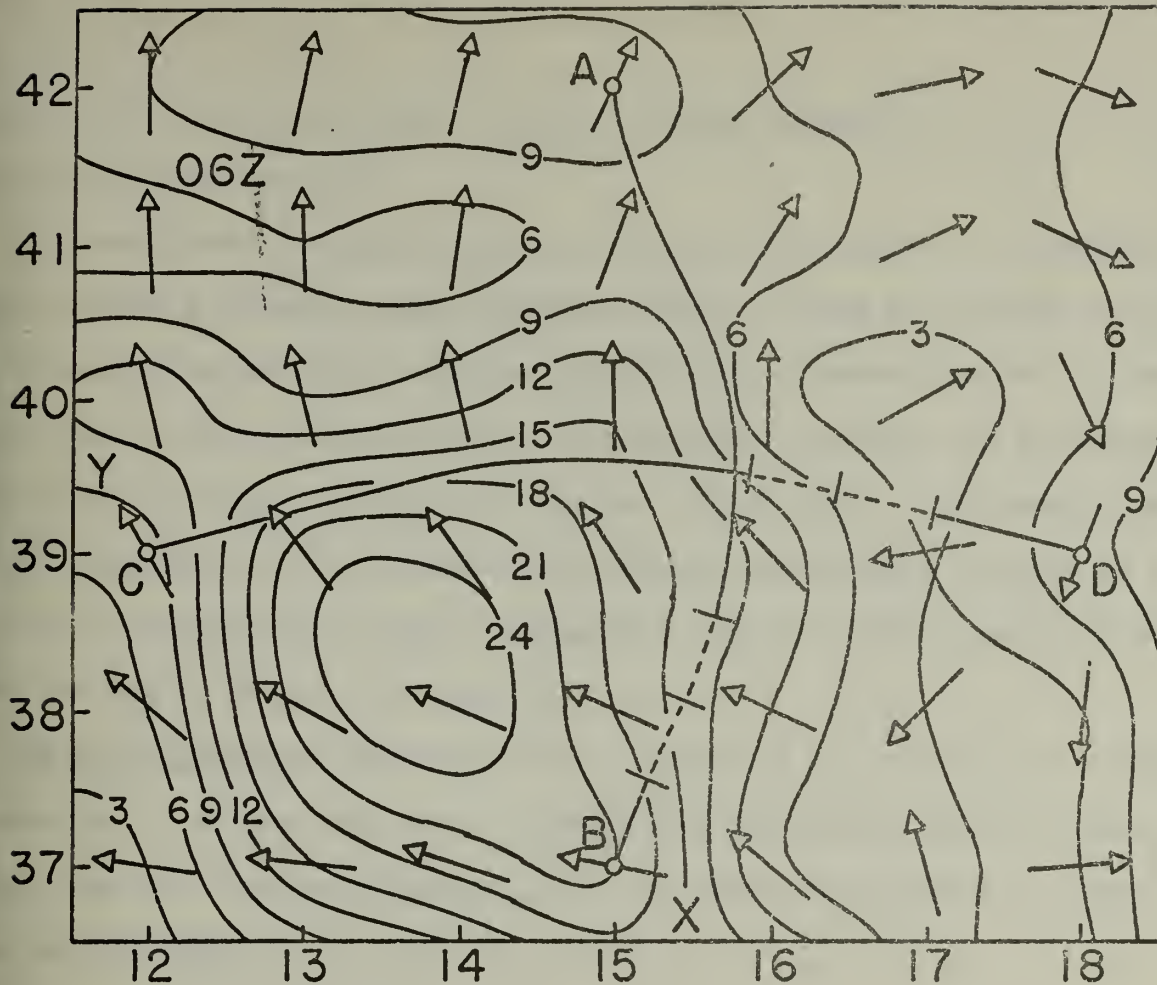


Fig. 6. Sea conditions at 06Z and 18Z on 6 May 1963.

sary when the wave height was more nearly analytic.

6. Concluding remarks

The numerical integrations involved in the theory of minimal-time ship routing through time-dependent wave fields are found to be feasible. The necessary three-dimensional interpolations in the wave field data, discussed in the Appendix, present no problem. Convergence problems may arise, but can be solved by the described delayed approach to the limit. The authors can supply copies of their Fortran programs for ship routing and for the cubic-interpolation contouring of Figs. 4, 5 and 6.

Acknowledgements. This work was supported by the Office of Naval Research. Aid from the U. S. Navy Fleet Numerical Weather Facility and from the Computer Facility, U. S. Naval Postgraduate School, is acknowledged.

7. Appendix

Some pertinent mathematical details are listed here.

The geographic wave direction ψ , measured clockwise from the North, must be converted to the unit vector $i \cos K + j \sin K$ in the stereographic grid system by

$$\begin{aligned}\cos K &= -[(x-31)\cos\psi + (y-31)\sin\psi]/r \\ \sin K &= [(x-31)\sin\psi - (y-31)\cos\psi]/r\end{aligned}\tag{23}$$

where $r^2 = (x-31)^2 + (y-31)^2$. Then the derivative

$$K_x = \cos K (\partial \sin K / \partial x) - \sin K (\partial \cos K / \partial x).\tag{24}$$

The derivatives $V_x = mv_x + vm_x$ and $V_p = mv_p$ are obtained most conveniently by the implicit differentiation of the equation

$$[v \sin(p-K)/b]^2 + [(c+v \cos(p-K))/a]^2 = 1\tag{25}$$

of the elliptical polar velocity diagram, and noting that a, b, c are functions of $H(x, y, t)$, and that K depends on x, y, t . The com-

plexity of the result is reduced by introducing the parameter β defined by

$$\begin{aligned}\sin\beta &= b \sin(q-K)/s = v \sin(p-K)/b \\ \cos\beta &= a \cos(q-K)/s = [v \cos(p-K) + c]/a\end{aligned}\quad (26)$$

where $s^2 = a^2 \cos^2(q-K) + b^2 \sin^2(q-K)$.

The numerical integration of the adjoint Eqs.(7) and (8) demands an interpolation formula for $H(x,y,t)$, $\cos K(x,y,t)$ and $\sin K(x,y,t)$ which guarantees the continuity of these functions and of their first space and time derivatives where any of x,y,t assume grid values. A 16-point interpolation formula to accomplish this is obtained from the 4×4 matrix F , whose four rows and columns of function entries correspond to four successive x and y grid values respectively. The interpolation mesh cell is the central cell of the array, with x and y measured from the cell center, and with the mesh distance considered to be two units. The formula is

$$F(x,y) = P(x) F P'(y) / 256 \quad (27)$$

where the matrix

$$P(x) = [(1-x)(x^2-1), (x-1)(3x^2+2x-9), (x+1)(9+2x-3x^2), (x+1)(x^2-1)] \quad (28)$$

and the prime indicates matrix transposition. Interpolation in the time dimension is accomplished by the similar 2-unit-mesh central-difference formula

$$F(t) = [F(-3), F(-1), F(1), F(3)] P'(t) / 16 \quad (29)$$

which guarantees the continuity of $F(t)$ and dF/dt at each end of the central time interpolation mesh. This formula is consistent with parabolic interpolation at the beginning or end of a time series, where central differences are not available. An interpolated vector $i \cos K(x,y,t) + j \sin K(x,y,t)$ should be normalized before use.

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- Haltiner, G. J., Hamilton, H. D. and Arnason, G., 1962: Minimal-time ship routing. J. Applied Meteor., 1, 1-7.
- Faulkner, F. D., 1963: Numerical methods for determining optimum ship routes. Navigation, 10, 351-367.
- James, R. W., 1957 (revised 1959): Application of wave forecasts to marine navigation. U. S. Navy Hydrographic Office.

FIGURE LEGENDS

- Fig. 1. Polar velocity diagram.
- Fig. 2. Ship speed in following, beam and head waves.
- Fig. 3. Ship motion in stereographic Oxy plane.
- Fig. 4. Sea conditions at 06Z and 18Z on 4 May 1963.
- Fig. 5. Sea conditions at 06Z and 18Z on 5 May 1963.
- Fig. 6. Sea conditions at 06Z and 18Z on 6 May 1963.


```

PROGRAM MINVOY
YVARS(1)=XLM1 YVARS(2)=XMU1 YVARS(3)=XLM2 YVARS(4)=XMU2 JJ=0 EW
YVARS(5)=X YVARS(6)=Y YVARS(7)=XJ YVARS(8)=S JJ=1 NS
DIMENSION KH(19,12,10),HT(19,12,10),KK(19,12,10),COSK(19,12,10),
+ SINK(19,12,10),YVARS(8),DY(8),YC(8),C(4),TAU(300),X(300),Y(300),
+ CAPLAM(300),PP(300),QQ(300),WK(300),VS(300),WH(300),XJ(300),
+ S(300),AK(4,8)
EQUIVALENCE (KH,HT), (KK,COSK)
READ 1, KH, KK, JJ, LMAX, KXST, KYST, KXFN, KYFN, ALF, T, FAC, FMUL
1 FORMAT ( 60(38I2/), 60(38I2/), 6I3, 4F12.9 )
PRINT 2, JJ, LMAX, KXST, KYST, KXFN, KYFN, FAC, FMUL
2 FORMAT (1H0, 6I3, 2F12.9)
DO 4 I=1,19
DELI = I-26
DO 4 J=1,12
DELJ = J+1
ROOT = SQRTF(DELI*DELI + DELJ*DELJ)
DO 4 K=1,10
HT(I,J,K) = KH(I,J,K)
ANGLE = KK(I,J,K) * 10
ANGLE = ANGLE/57.2957 7951
COS = COSF(ANGLE)
SIN = SINF(ANGLE)
COSK(I,J,K) = -(DELI*COS + DELJ*SIN)/ROOT
4 SINK(I,J,K) = (DELI*SIN - DELJ*COS)/ROOT
C(1) = 0.0
C(2) = 0.5
C(3) = 0.5
C(4) = 1.0
X(1) = KXST
Y(1) = KYST
TAU(1) = 0.0
XFIN = KXFN
YFIN = KYFN
WH(1) = HT(KXST,KYST,1)
CALL POLAR (COSK(KXST,KYST,1),SINK(KXST,KYST,1),WK(1))
XSTEP = T/FAC
CALL ABC (WH(1),AI,BI,CI,DAI,DBI,DCI)
CAPLAM(1) = 1.0
S(1) = 0.0
XJ(1) = 0.0
DO 18 L=1,LMAX
Q = ALF
QQ(1) = Q * 57.2957 7951
COSA = COSF(ALF)
SINA = SINF(ALF)
COSQ = COSA
SINQ = SINA
COSQMK = COSQ*COSK(KXST,KYST,1) + SINQ*SINK(KXST,KYST,1)
SINQMK = SINQ*COSK(KXST,KYST,1) - COSQ*SINK(KXST,KYST,1)
ABS = AI*COSQMK
ORD = BI*SINQMK
HYP = SQRTF(ABS*ABS + ORD*ORD)
SINB = ORD/HYP
COSB = ABS/HYP
VMAJ = AI*COSB - CI
VMIN = BI*SINB
VS(1) = SQRTF(VMAJ*VMAJ + VMIN*VMIN)
CALL POLAR (VMAJ,VMIN,PMK)
PP(1) = PMK + WK(1)
XVAR = 0.0
DO 5 I=1,8
5 YVARS(I) = 0.0
YVARS(1) = 1.0
YVARS(4) = 1.0
YVARS(5) = X(1)
YVARS(6) = Y(1)
N1 = T/XSTEP + 1.0
XN1 = N1
STEP = T/XN1
N2 = N1 + 1
DO 14 K=2,N2
DO 7 I=1,4
XC = XVAR + C(I)*STEP
DO 6 J=1,8
6 YC(J) = YVARS(J) + C(I)*AK(I-1,J)
XLAM = YC(1)*COSA + YC(3)*SINA
XMU = YC(2)*COSA + YC(4)*SINA
CLAM = SQRTF(XLAM*XLAM + XMU*XMU)
CALL TERP (HT,YC(5),YC(6),XC,H,HX,HY)

```


CALL	ABC	(H,FA,FB,FC,DA,DB,DC)	81
CALL	TERP	(COSK,YC(5),YC(6),XC,CK,CKX,CKY)	82
CALL	TERP	(SINK,YC(5),YC(6),XC,SK,SKX,SKY)	83
ROOT	=	SQRTF(CK*CK + SK*SK)	84
CK	=	CK/ROOT	85
SK	=	SK/ROOT	86
DKX	=	CK*SKX - SK*CKX	87
DKY	=	CK*SKY - SK*CKY	88
COSQ	=	XLAM/CLAM	89
SINQ	=	XMU/CLAM	90
COSQMK	=	COSQ*CK + SINQ*SK	91
SINQMK	=	SINQ*CK - COSQ*SK	92
ABS	=	FA*COSQMK	93
ORD	=	FB*SINQMK	94
HYP	=	SQRTF(ABS*ABS + ORD*ORD)	95
SINB	=	ORD/HYP	96
COSB	=	ABS/HYP	97
VMAJ	=	FA*COSB - FC	98
VMIN	=	FB*SINB	99
V	=	SQRTF(VMAJ*VMAJ + VMIN*VMIN)	100
COSP	=	(CK*VMAJ - SK*VMIN)/V	101
SINP	=	(SK*VMAJ + CK*VMIN)/V	102
COST	=	VMAJ/V	103
VBR	=	V/18.702 181 818	104
DELX	=	YC(5) - 26.0	105
DELY	=	YC(6) + 1.0	106
EMFI	=	(973.75 + DELX*DELX + DELY*DELY)/1043.638 743	107
CAPV	=	VBR*EMFI	108
DY(5)	=	CAPV*COSP	109
DY(6)	=	CAPV*SINP	110
EMFIX	=	DELX/521.819 3715	111
EMFIY	=	DELY/521.819 3715	112
DY(8)	=	CAPV	113
B2MA2	=	FB*FB - FA*FA	114
AMCCB	=	FA - FC*COSB	115
RAT	=	SINB*(FA*FC + B2MA2*COSB)/(FB*AMCCB)	116
VP	=	RAT * V	117
CAPVP	=	RAT * CAPV	118
DIV	=	1.+RAT**2-(V/AMCCB)**2*((VP*COST/SINB)-(B2MA2*SINB**2/FB))/FB	119
DET	=	YC(1)*YC(4) - YC(2)*YC(3)	120
RADIX	=	SQRTF(1.0 + RAT*RAT)	121
DY(7)	=	(CAPV*DET*DET/DIV) * (RADIX/CLAM)**3	121.5
FNUM	=	-RAT*AMCCB	122
DBDH	=	DA*COSB**2 + FA*DB*SINB**2/FB - DC*COSB	123
RATX	=	(FNUM*DKX + HX*DBDH)/AMCCB	124
RATY	=	(FNUM*DKY + HY*DBDH)/AMCCB	125
VBRX	=	RATX*VBR	126
VBRY	=	RATY*VBR	127
CAPVX	=	VBRX*EMFI + VBR*EMFIX	128
CAPVY	=	VBRY*EMFI + VBR*EMFIY	129
DY(1)	=	-CAPVX*(YC(1)*COSP + YC(2)*SINP)	130
DY(2)	=	-CAPVY*(YC(1)*COSP + YC(2)*SINP)	131
DY(3)	=	-CAPVX*(YC(3)*COSP + YC(4)*SINP)	132
DY(4)	=	-CAPVY*(YC(3)*COSP + YC(4)*SINP)	133
DO 7	J=1,8		134
7	AK(I,J) = STEP * DY(J)		135
DO 8	J=1,8		136
8	YVARS(J) = YVARS(J) + (AK(1,J)+2.*AK(2,J)+2.*AK(3,J)+AK(4,J))/6.		137
	XVAR = XVAR + STEP		138
	TAU(K) = TAU(K-1) + STEP		139
	X(K) = YVARS(5)		140
	Y(K) = YVARS(6)		141
	IF (N2-K) 80,81,80		141.0
80	IF (LMAX-L) 82,81,82		141.1
81	XLAM = YVARS(1)*COSA + YVARS(3)*SINA		142
	XMU = YVARS(2)*COSA + YVARS(4)*SINA		143
	CLAM = SQRTF(XLAM*XLAM + XMU*XMU)		144
	CAPLAM(K) = CLAM		145
	CALL TERP (HT,X(K),Y(K),TAU(K),WH(K),HX,HY)		146
	CALL ABC (WH(K),FA,FB,FC,DA,DB,DC)		147
	CALL TERP (COSK,X(K),Y(K),TAU(K),CK,CKX,CKY)		148
	CALL TERP (SINK,X(K),Y(K),TAU(K),SK,SKX,SKY)		149
	ROOT = SQRTF(CK*CK + SK*SK)		150
	CK = CK/ROOT		151
	SK = SK/ROOT		152
	COSQ = XLAM/CLAM		153
	SINQ = XMU/CLAM		154
	COSQMK = COSQ*CK + SINQ*SK		155
	SINQMK = SINQ*CK - COSQ*SK		156
	ABS = FA*COSQMK		157
	ORD = FB*SINQMK		158

	HYP = SQRTF(ABS*ABS + ORD*ORD)	159
	SIN8 = ORD/HYP	160
	COS8 = ABS/HYP	161
	VMAJ = FA*COS8 - FC	162
	VMIN = FB*SIN8	163
	VS(K) = SQRTF(VMAJ*VMAJ + VMIN*VMIN)	164
	COSP = (CK*VMAJ - SK*VMIN)/VS(K)	165
	SINP = (SK*VMAJ + CK*VMIN)/VS(K)	166
	CALL POLAR (COSP, SINP, PP(K))	167
	CALL POLAR (COSQ, SINQ, QQ(K))	168
	CALL POLAR (CK, SK, WK(K))	169
	XJ(K) = YVARS(7)/(YVARS(1)*YVARS(4) - YVARS(2)*YVARS(3))	170
	S(K) = YVARS(8)	171
	IF (N2-K) 82,15,82	171.1
82	IF (JJ) 88,83,88	172
83	IF (XFIN-X(1)) 85,21,84	173 EW
84	IF (X(K)-XFIN) 86,11,11	173.1E
85	IF (X(K)-XFIN) 11,11,86	173.2W
86	IF (Y(K)-3.00) 11,11,87	173.3
87	IF (10.0-Y(K)) 11,11,14	173.4
88	IF (YFIN-Y(1)) 90,21,89	174 NS
89	IF (Y(K)-YFIN) 91,11,11	174.1N
90	IF (Y(K)-YFIN) 11,11,91	174.2S
91	IF (X(K)-3.00) 11,11,92	174.3
92	IF (16.0-X(K)) 11,11,14	174.4
11	N2 = K	175
12	T = TAU(K)	176
	PRINT 13, N2	177
13	FORMAT (12H0 EARLY N2= 12/)	178
	GO TO 81	179
14	CONTINUE	180
15	PRINT 16	181
16	FORMAT (1H06X2HN212X3HALF14X1HT14X1HX13X1HY)	182
	PRINT 17, N2, ALF, T, X(N2), Y(N2)	183
17	FORMAT (110, 4F15.9)	184
	DELX = X(N2) - 26.0	185
	DELY = Y(N2) + 1.0	186
	EMFI = (973.75 + DELX*DELX + DELY*DELY)/1043.638 743	187
	CAPV = VS(N2)*EMFI/18.702 181 818	188
	XDOT = CAPV*COSP	189
	YDOT = CAPV*SINP	190
	FINJ = XJ(N2)	191
	DETER = FINJ*(XDOT*XLAM + YDOT*XMU)	192
	DIFX = XFIN - X(N2)	193
	DIFY = YFIN - Y(N2)	194
	DIFT = FINJ*(XLAM*DIFX + XMU*DIFY)/DETER	195
	DIFA = (XDOT*DIFY - YDOT*DIFX)/DETER	196
	T = T + FMUL*DIFT	197
	ALF = ALF + FMUL*DIFA	198
	PRINT 16	199
	PRINT 17, N2, ALF, T, X(N2), Y(N2)	200
18	CONTINUE	201
	PRINT 19	202
19	FORMAT(1H04X3HTAU8X1HX9X1HY6X6HCAPLAM5X2HWH8X2HWK8X2HVS9X1HS8X2HXJ	203
	+8X2HPP8X2HQQ/)	204
	PRINT 20, (TAU(I), X(I), Y(I), CAPLAM(I), WH(I), WK(I),	205
	+ VS(I), S(I), XJ(I), PP(I), QQ(I), I=1, N2)	206
20	FORMAT (11F10.4)	207
21	STOP	208
	END	209
	SUBROUTINE ABC (H, A, B, C, DA, DB, DC)	0
	RAD1 = SQRTF((0.038 635 06466*H-0.492 435 9727)*H+2.099 340 872)	1
	A = -0.120 197 666*H + 20.748 910 236 - RAD1	2
	RAD2 = SQRTF((0.032 937 36384*H-0.439 806 7919)*H+1.778 122 940)	3
	B = -0.126 879 519*H + 20.633 462 763 - RAD2	4
	RAD3 = SQRTF((0.010 441 8956*H-0.068 479 3891)*H+0.371 204 6341)	5
	RAD4 = SQRTF((0.013 051 7910*H-0.193 791 6314)*H+0.814 886 3262)	6
	C = 0.125 424 493*H - 0.293 444 8925 - RAD3 + RAD4	7
	DA = -0.120 197 666 -(0.038 635 06466*H - 0.246 217 98635)/RAD1	8
	DB = -0.126 879 519 -(0.032 937 36384*H - 0.219 903 39595)/RAD2	9
	DC = 0.125 424 493 -(0.010 441 89560*H - 0.034 239 69455)/RAD3	10
	+ (0.013 051 79100*H - 0.096 895 81570)/RAD4	11
	END	12
	SUBROUTINE POLAR (X, Y, P)	1
	IF (X) 10, 3, 10	2
3	IF (Y) 8, 4, 6	3
4	P= 0.0	4
5	RETURN	5
6	P=90.0	6
7	RETURN	7
8	P= -90.0	8


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9 RETURN
0 P= 57.2957 7951 * ATANF(Y/X)
1 IF (X) 12,14,14
2 IF (Y) 15,13,13
3 P= P + 180.0
4 RETURN
5 P= P - 180.0
6 END
SUBROUTINE TERP (FUNC, X, Y, T, OUT, OUTX, OUTY)
DIMENSION FUNC(19,12,10),PT(4,4),P(4),Q(4),PX(4),QY(4),ORD(4),
+ ORDX(4),ORDY(4),S(4),D(4)
LL = 1
L = XINTF(T)
IF (L) 1,4,1
1 IF (8-L) 3,2,3
2 LL = 3
L = L-2
GO TO 4
3 LL = 2
L = L-1
4 M = XINTF(X) - 2
N = XINTF(Y) - 2
XX = 2.0*(X-INTF(X)) - 1.0
YY = 2.0*(Y-INTF(Y)) - 1.0
TT = 2.0*(T-INTF(T)) - 1.0
XP1= XX + 1.0
XM1= XX - 1.0
YP1= YY + 1.0
YM1= YY - 1.0
TP1= TT + 1.0
TM1= TT - 1.0
X2M= XP1*XM1
Y2M= YP1*YM1
T2M= TP1*TM1
P(1) = -XM1*X2M
P(2) = ((3.*XX+2.)*XX-9.)*XM1
P(3) = -2.*XX*XX + 18. - P(2)
P(4) = XP1*X2M
Q(1) = -YM1*Y2M
Q(2) = ((3.*YY+2.)*YY-9.)*YM1
Q(3) = -2.*YY*YY + 18. - Q(2)
Q(4) = YP1*Y2M
PX(4)= (3.*XX-1.)*XP1
PX(1)= 4.*XX - PX(4)
PX(2)= (9.*XX-11.)*XP1
PX(3)= -4.*XX - PX(2)
QY(4)= (3.*YY-1.)*YP1
QY(1)= 4.*YY - QY(4)
QY(2)= (9.*YY-11.)*YP1
QY(3)= -4.*YY - QY(2)
DO 6 K=1,4
DO 5 I=1,4
DO 5 J=1,4
5 PT(I,J) = FUNC(M+I,N+J,L+K)
DO 10 I=1,4
S(I) = P(1)*PT(1,I)+ P(2)*PT(2,I)+ P(3)*PT(3,I)+ P(4)*PT(4,I)
D(I) = Q(1)*PT(1,I)+ Q(2)*PT(2,I)+ Q(3)*PT(3,I)+ Q(4)*PT(4,I)
ORD(K) = (D(1)*P(1) + D(2)*P(2) + D(3)*P(3) + D(4)*P(4))/256.
ORDX(K)= (D(1)*PX(1) + D(2)*PX(2) + D(3)*PX(3) + D(4)*PX(4))/128.
6 ORDY(K)= (S(1)*QY(1) + S(2)*QY(2) + S(3)*QY(3) + S(4)*QY(4))/128.
IF (LL-2) 8,7,9
7 G = (3.0*TT+2.0)*TT - 9.0
H = G - 4.0*TT
OUT = (TM1*(G*ORD( 2)-T2M*ORD( 1))-TP1*(H*ORD( 3)-T2M*ORD( 4)))/16.
OUTX= (TM1*(G*ORDX(2)-T2M*ORDX(1))-TP1*(H*ORDX(3)-T2M*ORDX(4)))/16.
OUTY= (TM1*(G*ORDY(2)-T2M*ORDY(1))-TP1*(H*ORDY(3)-T2M*ORDY(4)))/16.
RETURN
8 TM3 = TT - 3.0
OUT = ( TM3*(TM1*ORD( 1)-2.0*TP1*ORD( 2)) + T2M*ORD( 3))/8.0
OUTX= ( TM3*(TM1*ORDX(1)-2.0*TP1*ORDX(2)) + T2M*ORDX(3))/8.0
OUTY= ( TM3*(TM1*ORDY(1)-2.0*TP1*ORDY(2)) + T2M*ORDY(3))/8.0
RETURN
9 TP3 = TT + 3.0
OUT = ( TP3*(TP1*ORD( 4)-2.0*TM1*ORD( 3)) + T2M*ORD( 2))/8.0
OUTX= ( TP3*(TP1*ORDX(4)-2.0*TM1*ORDX(3)) + T2M*ORDX(2))/8.0
OUTY= ( TP3*(TP1*ORDY(4)-2.0*TM1*ORDY(3)) + T2M*ORDY(2))/8.0
END
END
6 10 10 10 5-1.362675810 4.97704031150.000000000 0.500000000

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TO CONVERT PROGRAM MINVOY TO PROGRAM STRAIGHT		
(1)	REPLACE MINVOY CARDS (43-55) BY FOLLOWING CARDS	43FS
	COST = -SINK(KXST, KYST, 1)	43FE
OR	COST = COSK(KXST, KYST, 1)	44FS
	SINT = -COSK(KXST, KYST, 1)	44FE
OR	SINT = -SINK(KXST, KYST, 1)	45F
	QUAD = BI*BI*COST*COST + AI*AI*SINT*SINT	46F
	RADI = SQRTF(QUAD - (CI*SINT)**2)	47F
	SINB = SINT*(AI*RADI - BI*CI*COST)/QUAD	48F
	COSB = (BI*COST*RADI + AI*CI*SINT*SINT)/QUAD	
(2)	REPLACE MINVOY CARDS (89-97) BY FOLLOWING CARDS	89FS
	COST = -SK	89FE
OR	COST = CK	90FS
	SINT = -CK	90FE
OR	SINT = -SK	91F
	QUAD = FB*FB*COST*COST + FA*FA*SINT*SINT	92F
	RADI = SQRTF(QUAD - (FC*SINT)**2)	93F
	SINB = SINT*(FA*RADI - FB*FC*COST)/QUAD	94F
	COSB = (FB*COST*RADI + FA*FC*SINT*SINT)/QUAD	
(3)	REPLACE MINVOY CARDS (153-161) BY FOLLOWING CARDS	153FS
	COST = -SK	153FE
OR	COST = CK	154FS
	SINT = -CK	154FE
OR	SINT = -SK	155F
	QUAD = FB*FB*COST*COST + FA*FA*SINT*SINT	156F
	RADI = SQRTF(QUAD - (FC*SINT)**2)	157F
	SINB = SINT*(FA*RADI - FB*FC*COST)/QUAD	158F
	COSB = (FB*COST*RADI + FA*FC*SINT*SINT)/QUAD	
(4)	REPLACE MINVOY CARDS (197-198) BY FOLLOWING CARDS	197FS
	FF = (5.00 - Y(N2-1))/(Y(N2) - Y(N2-1))	197FE
OR	FF = (13.0 - X(N2-1))/(X(N2) - X(N2-1))	198F
	T = FF*TAU(N2) + (1.0 - FF)*TAU(N2-1)	
PROGRAM CONTOUR		0
DIMENSION ABS(900), ORD(900), KH(19, 12), HT(19, 12), KK(19, 12), PT(12),		1
+ COSK(19, 12), SINK(19, 12), RX(48), RY(48), CON(4, 4), IT(12), TI(12), D(3)		2
EQUIVALENCE (IT, TI), (LA, AL)		3
READ 1, DS, IHMAX, IDELH, IXMIN, IXMAX, IYMIN, IYMAX, KH, KK, TI, AL		4
1	FORMAT (F4.2, 6I3/, 12(38I2/), 2(10A8/))	5
	DO 2 J=1, 12	6
	DO 2 I=1, 19	7
2	HT(I, J) = KH(I, J)	8
	PRINT 3, DS, IHMAX, IDELH, IXMIN, IXMAX, IYMIN, IYMAX	9
3	FORMAT (1H0, F4.2, 6I3)	10
	READ 26, (ABS(I), ORD(I), I=1, 5)	11
26	FORMAT (10F3.1)	12
	CALL CRAW (5, ABS, ORD, 1, 0, LA, IT, 1., 1., 0, 0, 2, 2, 8, 8, 0, LAST)	13
	C = DS*DS	14
	DO 22 IY= IYMIN, IYMAX	15
	Y = IY	16
	DO 22 IX= IXMIN, IXMAX	17
	X = IX	18
	CALL COEF (HT, IX, IY, PT, CON)	19
	DO 22 IH= 3, IHMAX, IDELH	20
	H = IH	21
	NR = 0	22
	CALL ROOT (CON, 1, H, D, KER)	23
	IF (KER) 8, 8, 4	24
4	DO 7 J=1, KER	25
	ABS(1) = X + (D(J)+1.0)/2.0	26
	ORD(1) = Y	27
	IF (NR) 33, 35, 33	28
33	DO 34 I=1, NR	29
	A = ABS(1) - RX(I)	30
	B = ORD(1) - RY(I)	31
	IF (A*A + B*B - C) 7, 7, 34	32
34	CONTINUE	33
35	CALL GRAD (PT, ABS(1), ORD(1), DX, DY, QUAD, DELX, DELY, DS, H, 0)	34
	IF (QUAD - 1.E-10) 7, 7, 5	35
5	IF (DELY) 6, 36, 36	41
6	DS = -DS	42
	DELX= DELX - DX - DX	43
	DELY= DELY - DY - DY	44
	IF (DELY) 7, 36, 36	45
36	ABS(2) = ABS(1) + DELX	46
	ORD(2) = ORD(1) + DELY	47
	CALL OUT (NR, ABS, ORD, RX, RY, DS, PT, X, Y, H)	48
7	CONTINUE	49
	IF (KER) 13, 13, 9	51
8	CALL ROOT (CON, 2, H, D, KER)	50
9	DO 12 J=1, KER	52


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ABS(1) = X
ORD(1) = Y + (D(J)+1.0)/2.0
IF (NR) 37,39,37
37 DO 38 I=1,NR
A = ABS(1) - RX(I)
B = ORD(1) - RY(I)
IF (A*A + B*B - C) 12,12,38
38 CONTINUE
39 CALL GRAD (PT,ABS(1),ORD(1),DX,DY,QUAD,DELX,DELY,DS,H,0)
IF (QUAD - 1.E-10) 12,12,10
10 IF (DELX) 11,40,40
11 DS = -DS
DELY= DELY - DY - DY
DELX= DELX - DX - DX
IF (DELX) 12,40,40
40 ABS(2) = ABS(1) + DELX
ORD(2) = ORD(1) + DELY
CALL CUT (NR,ABS,ORD,RX,RY,DS,PT,X,Y,H)
12 CONTINUE
13 CALL ROOT (CON, 3, H, D, KER)
IF (KER) 18,18,14
14 DO 17 J=1,KER
ABS(1) = X + (D(J)+1.0)/2.0
ORD(1) = Y + 1.0
IF (NR) 41,43,41
41 DO 42 I=1,NR
A = ABS(1) - RX(I)
B = ORD(1) - RY(I)
IF (A*A + B*B - C) 17,17,42
42 CONTINUE
43 CALL GRAD (PT,ABS(1),ORD(1),DX,DY,QUAD,DELX,DELY,DS,H,0)
IF (QUAD - 1.E-10) 17,17,15
15 IF (DELY) 44,44,16
16 DS = -DS
DELX= DELX - DX - DX
DELY= DELY - DY - DY
IF (DELY) 44,44,17
44 ABS(2) = ABS(1) + DELX
ORD(2) = ORD(1) + DELY
CALL CUT (NR,ABS,ORD,RX,RY,DS,PT,X,Y,H)
17 CONTINUE
18 CALL ROOT (CON, 4, H, D, KER)
IF (KER) 22,22,19
19 DO 22 J=1,KER
ABS(1) = X + 1.0
ORD(1) = Y + (D(J)+1.0)/2.0
IF (NR) 45,47,45
45 DO 46 I=1,NR
A = ABS(1) - RX(I)
B = ORD(1) - RY(I)
IF (A*A + B*B - C) 22,22,46
46 CONTINUE
47 CALL GRAD (PT,ABS(1),ORD(1),DX,DY,QUAD,DELX,DELY,DS,H,0)
IF (QUAD - 1.E-10) 22,22,20
20 IF (DELX) 48,48,21
21 DS = -DS
DELY= DELY - DY - DY
DELX= DELX - DX - DX
IF (DELX) 48,48,22
48 ABS(2) = ABS(1) + DELX
ORD(2) = ORD(1) + DELY
CALL CUT (NR,ABS,ORD,RX,RY,DS,PT,X,Y,H)
22 CONTINUE
DO 23 I=1,19
DELI = I-26
DO 23 J=1,12
DELJ = J+1
RAD = SQRTF(DELI*DELI + DELJ*DELJ)
ANGLE = KK(I,J)*10
ANGLE = ANGLE/57.2957 7951
COS = COSF(ANGLE)
SIN = SIN(ANGLE)
COSK(I,J) = -(DELI*COS + DELJ*SIN)/RAD
23 SINK(I,J) = (DELI*SIN - DELJ*COS)/RAD
IXP = IXMIN + 1
IYP = IYMIN + 1
DO 24 IY = IYP, IYMAX
Y = IY
DO 24 IX = IXP, IXMAX
X = IX
COS = COSK(IX,IY)

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SIN = SINK(IX,IY)
ABS(1) = X + COS/3.0
ORD(1) = Y + SIN/3.0
ABS(2) = X + X - ABS(1)
ORD(2) = Y + Y - ORD(1)
ABS(3) = ABS(2) + (1.732050807*COS - SIN)/18.0
ORD(3) = ORD(2) + (1.732050807*SIN + COS)/18.0
ABS(4) = ABS(3) + SIN/9.0
ORD(4) = ORD(3) - COS/9.0
ABS(5) = ABS(2)
ORD(5) = ORD(2)
DO 28 I=1,5
  ABS(I) = 0.8*ABS(I) - 4.0
  ORD(I) = 0.8*ORD(I) - 3.2
  CALL CRAW (5,ABS,ORD,2,0,LA,IT,1.,1.,0,0,2,2,8,8,0, LAST)
28 CONTINUE
  READ 29, (ABS(I), I=1,52)
  FORMAT ( 4(13F6.3/))
  READ 29, (ORD(I), I=1,52)
  DO 30 I=1,52
    ABS(I) = 0.8*ABS(I) - 4.0
    ORD(I) = 0.8*ORD(I) - 3.2
    CALL CRAW (52,ABS,ORD,2,0,LA,IT,1.,1.,0,0,2,2,8,8,0, LAST)
    READ 29, (ABS(I), I=1,52)
    READ 29, (ORD(I), I=1,52)
  DO 32 I=1,52
    ABS(I) = 0.8*ABS(I) - 4.0
    ORD(I) = 0.8*ORD(I) - 3.2
    CALL CRAW (52,ABS,ORD,3,0,LA,IT,1.,1.,0,0,2,2,8,8,0, LAST)
  32 STOP 25
25 END
SUBROUTINE COEF (HT, IX, IY, P, CON)
DIMENSION HT(19,12), P(12), CON(4,4)
P(1) = HT(IX,IY-1)
P(2) = HT(IX+1,IY-1)
LX = IX-2
DO 1 I=1,4
  P(2+I) = HT(LX+I,IY)
  P(6+I) = HT(LX+I,IY+1)
  P(11) = HT(IX,IY+2)
  P(12) = HT(IX+1,IY+2)
  CON(1,1) = P(6) - P(3) + 3.0*(P(4)-P(5))
  CON(2,1) = P(3) - P(4) - P(5) + P(6)
  CON(3,1) = P(3) - P(6) + 11.*(P(5)-P(4))
  CON(4,1) = -P(3) - P(6) + 9.0*(P(5)+P(4))
  CON(1,3) = P(10) - P(7) + 3.0*(P(8)-P(9))
  CON(2,3) = P(10) - P(9) - P(8) + P(7)
  CON(3,3) = P(7) - P(10) + 11.*(P(9)-P(8))
  CON(4,3) = -P(7) - P(10) + 9.0*(P(9)+P(8))
  CON(1,2) = P(11) - P(1) + 3.0*(P(4)-P(8))
  CON(2,2) = P(1) - P(4) - P(8) + P(11)
  CON(3,2) = P(1) - P(11) + 11.*(P(8)-P(4))
  CON(4,2) = -P(1) - P(11) + 9.0*(P(8)+P(4))
  CON(1,4) = P(12) - P(2) + 3.0*(P(5)-P(9))
  CON(2,4) = P(12) - P(9) - P(5) + P(2)
  CON(3,4) = P(2) - P(12) + 11.*(P(9)-P(5))
  CON(4,4) = -P(2) - P(12) + 9.0*(P(9)+P(5))
END
SUBROUTINE ROOT (CON, KS, H, D, KER)
DIMENSION CON(4,4), D(3), X(3)
FOURTH = CON(4,KS) - 16.0*H
IF (CON(1,KS)) 12,2,12
IF (CON(2,KS)) 6,3,6
IF (CON(3,KS)) 11,4,11
IF (FOURTH) 9,5,9
D(1) = -1.0
D(2) = 1.0
KER = 2
RETURN
6 B = 0.5*CON(3,KS)/CON(2,KS)
C = FOURTH/CON(2,KS)
QUAD = B*B - C
RAD = SQRTF(ABSF(QUAD))
IF (QUAD) 7,8,10
IF (RAD - 1.E-5) 8,8,9
8 X(1) = -B
KER = 1
GO TO 28
9 KER = 0
RETURN
10 X(1) = -B + RAD

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X(2) = -B - RAD
KER = 2
GO TO 28
11 X(1) = -FOURTH/CON(3,KS)
KER = 1
GO TO 28
12 A = CON(2,KS)/CON(1,KS)
B = CON(3,KS)/CON(1,KS)
C = FOURTH/CON(1,KS)
P = A*A/9.0 - B/3.0
Q = A*B/6.0 - C/2.0 - A*A*A/27.0
RAD = SQRTF(ABSF(P))
IF (P) 13,16,18
13 PHI3 = ASINH(Q/(RAD*RAD*RAD))/3.0
X(1) = 2.0*RAD*SINH(PHI3) - A/3.0
KER = 1
IF (1.732050807*RAD*COSH(PHI3) - 1.E-5) 15,15,28
15 X(2) = -(X(1)+A)/2.0
KER = 2
GO TO 28
16 X(1) = (2.0*ABSF(Q))*3.333333333333333 - A/3.0
KER = 1
IF (Q) 17,28,28
17 X(1) = -X(1) - A/1.5
GO TO 28
18 ARG = Q/(RAD*RAD*RAD)
IF (ABSF(ARG) - 1.0) 27,25,19
19 PHI3 = ACOSH(ABSF(ARG))/3.0
IF (Q) 21,24,24
21 X(1) = -2.0*RAD*COSH(PHI3) - A/3.0
22 KER = 1
IF (1.732050807*RAD*SINH(PHI3) - 1.E-5) 15,15,28
24 X(1) = 2.0*RAD*COSH(PHI3) - A/3.0
GO TO 22
25 X(1) = 2.0*ABSF(Q))*3.333333333333333 - A/3.0
X(2) = -(X(1)+A)/2.0
KER = 2
IF (Q) 26,28,28
26 X(2) = -X(2) - A/1.5
GO TO 17
27 PHI3 = ACOSF(ARG)/3.0
SIN = 1.732050807*RAD*SINF(PHI3)
Z = RAD*COSF(PHI3)
X(1) = Z + Z - A/3.0
X(2) = -Z + SIN - A/3.0
X(3) = X(2) - SIN - SIN
KER = 3
28 JAR = 0
DO 30 I=1,KER
IF (ABSF(X(I)) - 1.0) 29,29,30
29 JAR = JAR + 1
D(JAR) = X(I)
30 CONTINUE
KER = JAR
END
SUBROUTINE GRAD (P, X, Y, DX, DY, QUAD, DELX, DELY, DS, H, KD)
DIMENSION P(12)
XX = 2.0*(X - INTF(X)) - 1.0
YY = 2.0*(Y - INTF(Y)) - 1.0
XP1= XX + 1.0
XM1= XX - 1.0
YP1= YY + 1.0
YM1= YY - 1.0
TX= 2.0*XX
TY= 2.0*YY
CIR = 3.0*(XX*XX+YY*YY) - 10.0
X2M = XP1*XM1
Y2M = YP1*YM1
SXP = 6.0*XX + 2.0
SXM = SXP - 4.0
SYP = 6.0*YY + 2.0
SYM = SYP - 4.0
QXP = 18.0*XX + 2.0
QXM = QXP - 4.0
QYP = 18.0*YY + 2.0
QYM = QYP - 4.0
Q1 = X2M*P(10) + Y2M*P(12) - (CIR-TX-TY)*P(9)
Q2 = X2M*P( 7) + Y2M*P(11) - (CIR+TX-TY)*P(8)
Q3 = X2M*P( 3) + Y2M*P( 1) - (CIR+TX+TY)*P(4)
Q4 = X2M*P( 6) + Y2M*P( 2) - (CIR-TX+TY)*P(5)
P1 = (TX*P(10) - SXM*P(9))*XP1

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P2 = (TX*P(7) - SXP*P(8))*XM1	26
P3 = (TX*P(3) - SXP*P(4))*XM1	27
P4 = (TX*P(6) - SXM*P(5))*XP1	28
P5 = (TY*P(12) - SYM*P(9))*YP1	29
P6 = (TY*P(2) - SYP*P(5))*YM1	
P7 = (TY*P(1) - SYP*P(4))*YM1	31
P8 = (TY*P(11) - SYM*P(8))*YP1	32
HX = (YP1*(Q1-Q2+P1-P2) + YM1*(Q3-Q4+P3-P4))/16.0	33
HY = (XP1*(Q1-Q4+P5-P6) + XM1*(Q3-Q2+P7-P8))/16.0	34
QUAD = HX*HX + HY*HY	35
IF (QUAD - 1.E-10) 3,3,2	36
2 IF (KC) 5,4,5	37
4 HXX = (YP1*(SXP*P(10)-QXP*P(9)-SXM*P(7)+QXM*P(8)) +	38
+ YM1*(SXM*P(3)-QXM*P(4)-SXP*P(6)+QXP*P(5)))/8.0	39
+ HYY = (XP1*(SYP*P(12)-QYP*P(9)-SYM*P(2)+QYM*P(5)) +	40
+ XM1*(SYM*P(1)-QYM*P(4)-SYP*P(11)+QYP*P(8)))/8.0	41
HXY = (P1-P4+P3-P2+P5-P8+P7-P6+Q1-Q2+Q3-Q4)/4.0	42
RAD = SQRTF(QUAD)	43
DX = DS*HY/RAD	44
DY = -DS*HX/RAD	45
SEC = 0.5*(HXX*DX*DX + HXY*DX*DY + HYY*DY*DY)/QUAD	46
DELX= DX - HX*SEC	47
DELY= DY - HY*SEC	48
RETURN	49
5 DH = H - (YP1*(XP1*Q1-XM1*Q2) + YM1*(XM1*Q3-XP1*Q4))/32.0	50
DX = CH*HX/QUAD	51
DY = CH*HY/QUAD	52
DELX = DH*DH/QUAD	53
3 END	54
SUBROUTINE OUT (NR,ABS,ORD,RX,RY,DS,PT,X,Y,H)	0
DIMENSION ABS(900),ORD(900),RX(48),RY(48),PT(12)	1
C = 2.0*DS*DS	1.5
NPT = 1	2
NR = NR + 1	3
RX(NR) = ABS(1)	4
RY(NR) = ORD(1)	5
GO TO 1	6
4 IF (40-NPT) 20,20,24	7
24 CALL GRAD (PT,ABS(NPT),ORD(NPT),DX,DY,QUAD,DELX,DELY,DS,H,0)	8
IF (QUAD - 1.E-10) 19,19,5	9
5 ABS(NPT+1) = ABS(NPT) + DELX	14
ORD(NPT+1) = ORD(NPT) + DELY	15
1 CALL GRAD(PT,ABS(NPT+1),ORD(NPT+1),DX,DY,QUAD,DELX,DELY,DS,H,1)	16
IF (QUAD - 1.E-10) 19,19,6	17
6 IF (DELX - C) 25,25,19	18
25 NPT = NPT + 1	19
ABS(NPT) = ABS(NPT) + DX	20
ORD(NPT) = ORD(NPT) + DY	20.1
NPT1= NPT	21
IF (ORD(NPT)-Y) 7,10,8	22
7 NPT1= NPT1 - 1	23
GO TO 10	24
8 IF (Y+1.0-ORD(NPT)) 9,10,10	25
9 NPT1= NPT1 - 1	26
10 IF (ABS(NPT1)-X) 11,14,12	27
11 NPT1= NPT1 - 1	28
GO TO 15	29
12 IF (X+1.0-ABS(NPT1)) 11,14,14	30
14 IF (NPT-NPT1) 15,4,15	31
15 NPT = NPT1	32
19 IF (NPT-1) 26,26,20	33
20 NR = NR + 1	34
RX(NR) = ABS(NPT)	35
RY(NR) = ORD(NPT)	36
DO 21 I=1,NPT	37
ABS(I) = 0.8*ABS(I) - 4.0	38
21 ORD(I) = 0.8*ORD(I) - 3.2	39
LA = 4H	40
CALL CRAW (NPT,ABS,ORD,2,0,LA,7,1.,1.,0,0,2,2,8,8,0,LAST)	41
RETURN	42
26 NR = NR - 1	43
END	44
END	45
0 21 3 6 13 4 10 ZEROTH	

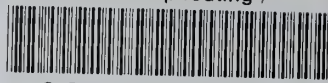
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7 AUG 68	17289
7 FEB 69	17980
27 NOV 70	18919
6 DEC 71	20553
13 JUN 74	S10519

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